MIE334: Numerical Methods Assignment 2

Due: February 25th, 11:59 pm, 2021

3. Yes, because the Frobenius norm of the matrix A is on the order of a thousand, meaning that the system is ill-conditioned and sensitive to round-off errors.  
     
   We can also see that in order to calculate in matrix L, we must divide by an element that is on the order of , meaning that will be on the order of . As a result, when we round off to the fifth decimal place, we can only keep one digit of precision. The other digits are lost, contributing to the inaccuracy of the result.

(2)

1. The solution will not converge because the matrix is not diagonally dominant. The diagonal elements of the matrix are less than the absolute row sum of the non-diagonal elements. To fix it, we can simply swap the columns of the coefficient matrix so that the matrix becomes diagonally dominant.  
     
   In this matrix, the diagonal elements are greater than the absolute row sum of the non diagonal elements. Therefore, the matrix will converge.

(b)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Iteration |  |  |  |  |  |  |
| 0 | 0 |  | 0 |  | 0 |  |
| 1 | -0.857143 | 100% | 0.535714 | 100% | 1.821428 | 100% |
| 2 | -1.040816 | 17.65% | 0.945153 | 43.32% | 1.988520 | 8.40% |
| 3 | -1.006195 | 3.44% | 0.995581 | 5.07% | 1.999556 | 0.55% |
| 4 | -1.005683 | 0.56% | 0.999748 | 0.42% | 2.0000107 | 0.02% |
| 5 | -1.000038 | 0.05% | 0.999993 | 0.03% | 2.000004 | 0.00% |

(c)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Iteration |  |  |  |  |  |  |
| 0 | 0 |  | 0 |  | 0 |  |
| 1 | -1.028572 | 100.00% | 0.591428 | 100.00% | 2.242286 | 100.000% |
| 2 | -1.10586122 | 6.99% | 1.122641633 | 47.32% | 2.021771755 | -10.91% |
| 3 | -0.96153578 | -15.01% | 0.993542467 | -12.99% | 1.985369791 | -1.83% |
| 4 | -1.00629181 | 4.45% | 0.9950149 | 0.15% | 2.002190365 | 0.84% |
| 5 | -0.99997172 | -0.63% | 1.001662614 | 0.66% | 2.000221316 | -0.10% |
| 6 | -0.99975858 | -0.02% | 0.999806299 | -0.19% | 1.999829972 | -0.02% |
| 7 | -1.00005234 | 0.03% | 0.999972029 | 0.02% | 2.000033286 | 0.01% |

The number of iterations needed when applying an overrelaxation factor is more than the number of iterations needed without the factor. The reason that it converges slower is because the overrelaxation factor is too large and it causes the variables to the correct values, thus increasing the number of iterations needed to be within the stopping criterion.

(d) Pseudocode:

|  |
| --- |
| X1 =0  X2 =0  X3 =0  Ea1=1  Ea2=1  Ea3=1  do:  X1\_old = X1  X2\_old = X2  X3\_old = X3  X1 = (-6+X2-X3)/7  X2 = (3+X1+X3)/4  X3 = (9-X1+2\*X2)/6  Ea1 = abs(X1\_old-X1)/X1  Ea2 = abs(X2\_old-X2)/X2  Ea3 = abs(X3\_old-X3)/X3  while(Ea1 and Ea2 and Ea3 all > 0.001) |

(3)